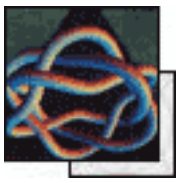
Analogy



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I. What is Analogy?

1. The Common Meaning of the Word “Analogy.” The word “analogy” in its usual sense in modern English means “a form of reasoning in which one thing is inferred to be similar to another thing in a certain respect, on the basis of the known similarity between the things in other respects” (*Random House Unabridged Dictionary* [Random House, Inc., 2006]). Recently, the adjectival form of the world “analogy,” “analog,” has come to be frequently used in a technical sense, denoting electronic devices that work in a way different from “digital” or “numerical” electronic devices. The origin of the word “analogy,” as the Greek root (*analoghía*) suggests, is ancient and is based on the mathematical concept of “proportion” ($a:b = c:d$), which establishes a similarity based on the equivalence of ratios. One could think, for instance, of the similarity of two triangles whose sides stand in a fixed ratio. The transfer of the word “analogy” from mathematics to logic and philosophy dates back to Plato (427-347 B.C.) who, however, never devised a theory of analogy. Aristotle (384-322 B.C.) was the first to give a systematic formulation of it in the field of logic. In the Middle Ages, Thomas Aquinas brought Aristotle’s work to perfection with both a philosophical and theological aim. Later, beginning with the Nominalists, analogy became less and less understood. It was gradually abandoned in the fields of logic and philosophy and restricted in its scope to the point of becoming a simple literary “metaphor.” It is in this sense that the term is used today in the context of hermeneutics.

2. Analogy and Logic. The need for introducing analogy into Greek thought seems to have arisen from two kinds of problems: the first was strictly "logical-linguistic," while the second one was more properly "metaphysical." From the logical-linguistic point of view, Aristotle, and later Thomas Aquinas, began with the observation that in common language—which expresses, and therefore is a sign of, the structure of how thought proceeds—the same term (or "predicate") can be attributed to different subjects in a "univocal," "equivocal," or "analogous" way. In the first case of univocity, the predicate has exactly the same meaning for the entire class of subjects to which it is attributed: For example, when we say, "Tom is a man" and "Dick is a man," the term "man" corresponds to the same definition "rational animal" in both instances. In the second case of equivocity, on the other hand, the same term is used with completely different and uncorrelated meanings, as when one says, "this animal is a bull" and "this document is a papal bull." In this case, the word "bull" corresponds to different definitions in each of the two examples. In the first example, it involves an "adult male bovine," in the second, "a text written by the Pope." Consequently, the use of the same word to signify different things is adopted purely by convention, so much so that equivocity is related to the language one uses and is lost in translation to another language. Finally, in the third case of analogy, the same term is used with different meanings but in such a way that they have a real correlation, and therefore the use of the same term indicates a real similarity and not a mere choice of convention. An example of this would be when one says "Einstein was clever" and "the theory of general relativity is clever." Properly speaking, only a man can be clever, but a theory can be said to be clever in so far as it is an expression and a "real effect" of the cleverness of its author (rather than a theory being considered clever merely by convention).

3. Analogy and Metaphysics. The second class of problems which have led to the idea of analogy is not purely logical and linguistic but more properly metaphysical, in that analogy is inherent in things and is successively transferred to the thought and language with which one attempts to understand reality. Greek thinkers confronted the problem of reconciling two seemingly contradictory facts of experience [2], namely, the being of things versus their "becoming" (or in physical terms, their "motion"). The "monistic" solution—that is, a solution based on the assumption that reality is founded on only one constitutive principle (be it material or immaterial)—requires that one take one of the two facts of experience as apparent: If one admits only the reality of being, as a single undifferentiated state, "being" can never be other than itself, as it cannot change into something different from itself. On the other hand, in adopting this approach, one cannot explain the phenomenon of motion that we observe in everyday experience, as the passage from one state to another. Therefore, one would have to say that this passage is not real but purely apparent (this was the solution proposed by Parmenides, 6th-5th century, B.C.). We are then left with the problem of understanding what produces this illusion in us. If, on the other hand, one only admits the reality of becoming, it is then necessary to admit the contradiction that becoming, by the very fact that it is, coincides with being, that multiplicity coincides with oneness, that nothingness (that is, non-being) is a state of being, and that becoming is a continuous oscillation between these two contradictory states. But, admitting this contradiction implies, in the end, that knowledge is impossible (this was the extreme consequence of Cratilus, following in the footsteps of Heraclitus, 6th-5th century B.C.). In order to explain human experience completely, it is necessary to hypothesize that being may exist according to "differentiated states" that constitute a spectrum of modes of being lying somewhere between being in its absolute fullness (God, Pure Act) and in its complete absence (nothingness). To correctly understand the analogy of being, we need the help of the accurate Latin terminology: *Ens* means "being" as a subject capable of being, while *esse* is the property of "being." Being (*esse*) is the principle by which a being (*ens*) is: "Being" (*ens*) is a term which is predicated in a differentiated but not equivocal way of different subjects.

The notion of analogy of being corresponds, from the logical point of view, to the metaphysical fact that assumes that being (*esse*) is actuated in differentiated modes and degrees in existing things (or, to say it

another way, that things participate in being to varying degrees). Thus, the logical theory of analogy corresponds to the metaphysical theory of participation.

II. Analogy in Aristotelian-Thomistic Logic and Metaphysics

In Aristotelian-Thomistic logic, three types of analogy are possible (although further distinctions have been introduced by later schools): analogy of "attribution" or "simple proportion," analogy of "proper" or "intrinsic proportionality," and analogy of "improper," or "extrinsic," or "metaphorical proportionality."

1. Analogy of Attribution or Simple Proportion. Analogy of attribution is usually presented with a classic example: "Tom is 'healthy,' his complexion is 'healthy,' this food is 'healthy,' the air is 'healthy.'" By observing this example, we note that the characteristic of being "healthy" is proper only to Tom, who is the only subject that can be said to enjoy good health, as he is the only living being of the things considered in this example. One cannot properly speak of the other things as being "healthy" because they are not living beings. One can say that in a certain sense these non-living beings are "healthy" only in reference to the good health of Tom, who alone is the subject of the predicate "health" in the proper sense. For this reason, Tom is called the *summum analogatum* or *primum analogatum*.

As for the other subjects, one can single out the relationship they have with the healthy state of being of Tom: His healthy complexion is a sign of his good health, in so far as it is an "effect" of his good health. Healthy food is that which favors Tom's good health as one of its "causes." It must be understood that the reference to the *summum analogatum* is neither conventional nor accidental, but is instead founded on reality and confirmed by experience (from the fact that healthy food really contributes to the good health of someone who eats such food, and that a healthy complexion is really a sign of good health, and so on and so forth). For this reason, food, complexion, and climate are referred to as the *analogata inferiora*. It is this reference, which is founded on reality, that makes the concept of attribution more than just "equivocal." These things and realities are and remain different, but the common name of the predicate expresses qualities which, even if they are in themselves different, have, under a certain aspect, a direct relationship with the quality of the *primum analogatum* (cf. Thomas Aquinas, *Summa Theologiae*, I, q. 13, a. 5).

2. Analogy of Proper or Intrinsic Proportionality. Even this second kind of analogy is usually illustrated with a classic example that consists in comparing sight with intelligence. We often use the idea of "vision" either in reference to "eyesight" or in reference to the "mind's understanding." Thus, we use the expressions, for example, "the light of truth illuminates the mind," "to understand at first glance," and "a philosophical vision of reality." In these examples, we have a term which expresses an action (seeing) which we attribute to two different subjects (the eye and the mind). In this type of analogy, the similarity is established between the "relations" between predicate and subjects rather than between different senses of the same predicate attributed to different subjects. This similarity between the relations can be summarized by a formula which recalls that of a mathematical proportion: "Seeing" is to the "eye" as "understanding" is to the "mind." Nevertheless, when we write a mathematical proportion, we establish two "equal" relations (2:3 = 4:6), whereas in the case of the analogy of proportionality, we state that two subject-predicate relations are not the same, but "similar" (cf. Thomas Aquinas, *De Veritate*, q. 2, a. 11). It must be emphasized that the action attributed to the subjects is really connected with each of them. The faculty of seeing is intrinsic to the eye, and the faculty of understanding is intrinsic to the mind: In both cases, we are dealing with a natural capacity, a proper and therefore really possessed faculty. For this reason, one speaks of analogy of "proper" or "intrinsic" proportionality. We note that in this type of

analogy there exists neither a *primum analogatum* nor *analogata inferiora*: We have instead a subject-quality relationship which can be applied, in the proper sense, to a subject (the eye in the case of vision) and in a "similar" sense to the other subject (the mind). Seeing is proper to the eye, not the mind. One can therefore say that, in a certain sense, what takes the place of the *primum analogatum* is not the subject to which the predicate is properly attributed, but a relation between the subject (the eye) and the predicate (able to see).

3. Analogy of Improper, Extrinsic, or Metaphoric Proportionality. The third type of analogy is that of the "metaphor." It involves a kind of analogy in which, unlike the two preceding cases, there is no real basis for similarity. It is a kind of analogy which is founded instead on a similarity discovered by the knowing subject who does not see any cause-effect relation in the nature of the subjects and the predicate, nor any real similarity in their relations. Properly speaking, it is not a real analogy, but we can consider it as such in a loose or improper sense. A typical example used to illustrate the concept of this kind of analogy is the following: "Tom has the courage of a lion." Even in this case there is implicitly a kind of proportion: We can, in fact, reformulate this example in the following terms: "Tom is as courageous as a lion is courageous." We see immediately that the quality "courageous" through which Tom can be likened to a lion is a quality that can be found in its highest degree in a lion: In a certain sense, this recalls the analogy of attribution. Nevertheless, there is a fundamental difference: There is no cause-effect relation between the courage of the lion and that of Tom, in that Tom is not courageous in virtue of a supposed participation in the courage of the lion. We cannot therefore speak of an analogy of proportion. It is instead a similarity that the knowing subject recognizes, as an external observer, between the courage of Tom and the courage of the lion. In this case, we have a similarity of relations between the subject and its quality, as in the case of the analogy of proportionality. Nevertheless, one cannot even speak of a true analogy of proper proportionality. In fact, in order to have an analogy of "proper" proportionality, the proportion that one wishes to establish would have to be: Tom is to the courage (of Tom) as the lion is to the courage (of the lion), whereas in the analogy of improper proportionality the same quality of courage proper to the lion (lion-like courage) is attributed to both Tom and the lion. Properly speaking, Tom has a human courage, while "lion-like courage" is attributed to him. We are dealing with a kind of "extrinsic" attribution, in that one attributes a character which is natural and proper to a lion to a natural endowment of Tom (cf. Thomas Aquinas, *Summa Theologiae*, I, q. 13, a. 3, 1um).

4. Analogia Entis. The fundamental discovery of the metaphysics of antiquity has probably been that of the analogy of being (*analogia entis*). Unlike the different *genera* which, from the logical point of view, are formalized in "universal" concepts predicated in a univocal way of various subjects, as "man" is said with the same meaning of Tom, Dick, and Harry—"being" is predicated in an analogous way of several subjects and rises above the *genera* and universal concepts which describe them (cf. Aristotle, *Metaphysics*, 998b, 22-27).

We note here two relevant aspects of the issue: First, in particular, "being" is said according to an analogy of proper proportionality of an object (substance) and its properties (accidents). This is a result of the fact that a property is always a property of something and can exist only in something else and not alone. A color, a length, a temperature, etc., exist always and only in an object, while an object possesses an autonomous existence. Thus, one must say that a property is to its mode of being as an object is to its mode of being, but the two modes are not identical, though they may have in common the fact of being. Second, in addition, "being" is said of a finite object, which has being by participation, and is said to be so according to an analogy of proportion with respect to Pure Act which is being in itself and is the cause of the being of a finite object. A similar property to that of "being" is also characteristic of the super-universal notions of "true," "one," and "good," which, together with "being," are called the "transcendentals."

5. The Crisis of Analogy. The concept of analogy, which finds its most complete development and use in the philosophy of Thomas Aquinas, contains, beginning with Thomas Aquinas' contemporaries, the seeds of its future downfall. In fact, from as early as the 13th century, the two great schools of philosophical-theological thought in Paris, where Albert the Great (1200-c.1280) and later his disciple Thomas Aquinas flowered, and in Oxford, with Roger Bacon (1214-1252), Robert Grosseteste (1174-1253), and later John Duns Scotus (1275-1308) and William of Ockham (1280-1349), were in opposition and would follow two different paths without ever coming to a mutual understanding. The Aristotelian path of Albertus Magnus and Thomas Aquinas would become of great importance especially for Catholic theology and, three centuries later, would be officially recognized in large part by the Council of Trent (1545-1563). The Platonic path, prevalent in Oxford, would concentrate on the problem of the mathematical formulation of the sciences, beginning with Roger Bacon, creating the methodological premises for the development of modern science.

In this way, there arose an ever more univocal and mathematical scientific way of thinking that took root and departed from a metaphysical and theological analogy-based thought. Duns Scotus would resolve the analogy of being in a multiplicity of univocals, just as William of Ockham would dissolve the reality of universals into pure names (Nominalism) by denying them a real existence outside of the mind. This development would then have an influence on the philosophical thought of Descartes (1596-1650), and later on Kant (1724-1805) and the success of Galilean and Newtonian science, and would eventually lead to the end of the very possibility of metaphysics as a science and consequently of theology as a systematic science. Nevertheless, in the last few decades, we have witnessed a new trend in the sciences which seem to be seeking, and in a certain sense discovering anew, the concept of analogy, with the aim of confronting new problems related to theories regarding the logical and mathematical foundations of the sciences and the complexity of self-organizing structures. Even if it is too early to judge, one could say that the concept of analogy, which was initially excluded from scientific thought for fear of equivocity, has now claimed its place. New disciplines like "formal ontology" seem to open up a new perspective, a sort of scientific approach to metaphysics. It is an approach that is claimed by modern mathematical logic and even by the technologies related to electronic computing and "artificial intelligence."

III. Analogy and Theology

Recourse to the concept of analogy in theology is necessary for many reasons. It cannot be otherwise since human reason, which is by its very nature creaturely, is able to approach the mystery of God only if it maintains a distance between creature and Creator by acknowledging that one can speak of God only by analogy and not in a univocal or equivocal way. In the context of the metaphysics of being, the *analogia entis* allows one to approach the problem of God's existence as the foundation of the being of all things and to predicate God's attributes and perfections that are present, in a participatory way, in God's works. But it is the very language of Revelation as presented in Sacred Scripture [3]which uses analogy in its various forms, be they proper or improper, as for example in metaphor and even in "parable," expressing, through human concepts, that which would otherwise remain transcendent and ineffable in itself. The language of analogy is then used by theologians in their attempts to approach, through recourse to images and comparisons, the mysteries of the faith, and it is also used in order to discern relations between such things, thereby grasping a deeper, inner coherence of God's plan of salvation.

1. The Knowledge of God and Divine Names. The various applications of the concept of analogy to theology lie on different levels. The first question one asks concerns the knowledge of God [4], either through human reason alone or by faith in what God has revealed about Himself. Theologians have

traditionally taken two paths to this goal. The first is the "apophatic" or "negative" way, typical of Eastern Christianity, which emphasizes the fact that we can only know with certainty what God is not, rather than what He is. Following this approach, such characteristics as composition, corporeality, finitude, and so on, are excluded from the notion of God. In addition to negative theology, and inspired by a scriptural passage from the Book of Wisdom (cf. *Wis* 13:5) in which explicit reference is made to the concept of analogy, Western Christianity developed a positive theology. On the basis of the analogy of simple proportion, it allows one to recognize in God a similarity with the perfections found in creation, as effects whose *summum analogatum* is God Himself (cf. Thomas Aquinas, *Summa Theologiae*, I, q. 12). This involves a cognitive approach which certainly does not dissolve mystery in that, as the Fourth Lateran Council (1215) recalled, "between Creator and creature, there is always a greater difference than likeness" (*DH* 806; *Fides et Ratio,* n. 19).

Another classical theological problem that is closely tied to the problem of the knowledge of God is that of the titles one can correctly attribute to God (the "divine names"). This theme, treated by pseudo-Dionysius in *De Divinis Nominibus*, was taken up and given a complete treatment by Thomas Aquinas for whom analogy would play a decisive role. First of all, he maintained that the names that denote what God most certainly is not (imperfections or ontological or moral limits) cannot be attributed to God. He then states that we can attribute to God the words we use to describe the perfections of creatures, but only by analogy, as our language refers mainly to what we know of creatures. These are in fact an effect of which God is the cause, a cause that cannot be known directly by us. We cannot speak of Him univocally because God is a cause that is infinitely higher than His effects and transcends their natures as He does not belong to any genus. We cannot speak of Him equivocally, since there is a cause-effect relation, which is a real relation *from the creatures* towards God. Thus, the names signifying God's perfections are used by analogy of proportion, God being here the *summum analogatum*. When one says that something is good, one says this most properly of God, who is good in and of Himself, rather than of creatures, who are good only by participation. Other names can be attributed to God only metaphorically. This happens either when one signifies a perfection by means of a name describing a creature who possesses it or when, instead of the name of a certain perfection, the creature's name is attributed to God, with the intention of attributing that perfection to Him. This happens, for example, when in Holy Scripture God is called a "rock" or "lion," with the intention of attributing the perfections of a rock and a lion to him (cf. *Summa Theologiae*, I, q. 13).

2. Examples of Analogy in the Scriptures. It is proper to the language of Holy Scripture to offer, through different literary genres, a treasure trove of analogies and metaphors. This is due, as already mentioned above, to the need for expressing with human words, which are used primarily to describe creatures, contents regarding the transcendent reality of God, which reason alone cannot reach and which is not an object of common experience. It is God who communicates His will and His plan through images based on analogy. Abraham is asked to try to conceive of the immense number of descendents of whom he is called to be the father by an analogy with the great number of stars in the sky and grains of sand in the sea (cf. *Gen* 15:5 and 22:17). Another example is the prophet Jeremiah, who is invited by God to look at the renewal that God will bring about in the house of Israel (*Jer* 18:1-4) by considering the analogy of the potter who forms and then destroys the work of his hands in order to make it anew. The prophets themselves were the ones who spoke to the people through numerous images and analogies, drawing from what happens in nature, in their own history, and in the story of different peoples (*Ez* 31:1-14; *Hos* 1:2-9; *Dan* 2:31-45).

Jesus spoke in "parables" rather frequently to describe the reality of the Kingdom with effective and coherent images, in order to make it more understandable to his audience. The expression "The *Kingdom of Heaven* is similar to" frequently recurs in the Gospels (cf. *Mt* 13:1-41; *Mk* 4:1-34; *Lk* 8:4-18). This

comparison is based on the "analogy of proportionality." The use of images and metaphors establishes a simile between a known reality and an unknown or difficult to understand one, allowing the transposition of properties and relations from the better known to the lesser known image. The parable is often told in the form of a story whose argumentative force consists in the narration of a fact (a fictitious but true-to-life fact) that the audience can understand well, and through which the audience can draw logical conclusions. Such conclusions, by dint of analogy, can be then applied to the initially unknown reality so as to understand some of its most important characteristics. The language of metaphor and parable, or if you prefer, of "narration," is particularly fitting to the human mind. By the use of it, we find ourselves in a situation in which it is possible to identify a series of unchanging relations between human beings and things, or between human beings themselves, that goes beyond the changing objects of experience. These relations can be used as logical, cosmological, and anthropological coordinates in order to communicate a certain message. It is not surprising that the Word of God, which has also taken on the history and logic of such communicative and cognitive structures (which were taken on together with the true humanity of Christ) makes recourse to it as a kind of "fundamental human language."

From a hermeneutic point of view, the language of analogy in Scripture has a special role, which must be distinguished from the symbolic one, which is also present. In the case of analogy an analogate is always referred to, whereas symbolic language refers to a reality beyond the limits of human discourse and language that requires completely new, non-analogous categories. But symbol remains incomplete without the help of analogy, since it recalls a reality independent of symbol itself, which carries the risk of mentally conceiving an infinite chain of symbols that never attains its real object.

3. Uses of Analogy in Theology. Analogies are widely used in Ecclesiology when speaking of the Church by resorting to "figures," as used for example by the Magisterium during the Second Vatican Council (cf. *Lumen Gentium*, 6). The mystery of the Church, in fact, participates in the richness and transcendence of God, since she has her origin in the mystery of God the Father's plan of salvation, and is revealed and accomplished through the missions of the Son and the Holy Spirit. In order to be expressed by words, the reality of the Church needs the analogy of intrinsic and extrinsic proportionality. Based on Sacred Scripture and the teachings of the Fathers of the Church, theology employs different images for the Church: a flock led by a shepherd, the Lord's vine, a house built on a keystone which is Christ, the Kingdom, the family and abode of God, and, above all, God's people and the Body of Christ. It should also be observed that one must use this last analogy not in a metaphorical, but in a proper, sense (cf. *Lumen Gentium*, 7; Pius XII, *Mystici Corporis*, June 29, 1943). The relationship between Christ and His Church is likened, in addition, to the relationship between bride and bridegroom, and also to the relationship of the head to its body. The peculiarity of such analogy-based images lies in the fact that none of them alone is adequate enough to express the mystery of the Church (she is visible and invisible, temporal and eternal, one, yet present in many places, distinct from her Bridegroom, and yet one with her Head, etc.), whereas all of them together play their parts in clarifying her character and properties.

Classical examples of the applications of analogy can be found in the teaching concerning the sacraments. As stages of the "Christian life," they can be compared to the various phases of "natural life," whether individual or social, according to an analogy of proper proportionality. In this way, Baptism is like the "birth" of Christian life, Confirmation is like "becoming an adult," the Eucharist is like nourishment for one's spiritual journey, and so on (cf. Thomas Aquinas, *Summa Theologiae*, III, q. 65). In the life of grace, then, sin is compared to death, so that one can understand its effects on the spiritual soul [5], in an analogy with what death brings about in the body. Even though such uses come with the limitations inherent in any type of comparison, they have undoubtedly aided our understanding of the mysteries of the faith and facilitated its diffusion.

Concerning the relationship between scientific thought and religious faith, the theological analogies used throughout history to clarify the relationship between faith and reason (or between philosophy and theology) are worthy of note. In medieval thought, philosophy is spoken of as the handmaiden of theology. Such a comparison, which has not infrequently been presented in a reductive and instrumental way, elicited an ironic response from Kant. Kant remarked that the handmaiden should have preceded her mistress, like a torch, in order to light the way. But the relationship between faith and reason has also been viewed as a marriage relationship (a typical image also used to describe the relationship between nature and grace, but one which stresses the greater dignity of the faith-husband pole). Contemporary theology in particular uses Marian and Christological analogies. For example, there is an analogy of the faith-word-Spirit that is accepted and embraced by an analogy of the reason-listening-Mary, thus "generating" the fruit of Theology (theology is used here in the strong sense of a wisdom which participates, by dint of Revelation, in the uncreated Wisdom of Christ). In a Christological analogy, reason and faith are seen in relation to each other as the human nature is seen in relation to the divine nature within the Person of the Divine Word made man (see Jesus Christ, Incarnation and doctrine of the Logos) [6]. As Christ's humanity gives visible and historical expression to the divine nature and person, so philosophy and reason give theology and faith an indispensable language to express, in a clearly limited and incomplete, but authentic, way that which one knows by faith as belonging to the transcendence of God.

Concerning the history of theology and its relationship with scientific thought, Joseph Butler's essay (1692-1752) titled *The Analogy of Natural and Revealed Religion in the Constitution and Course of Nature* (1736) must be mentioned. In it, the author presents the course of nature and of human history as a great analogy for the purpose of understanding the language and meaning of Christian revelation. This work became famous for its great influence on the thought of John Henry Newman (1801-1890) who often cited it in his books.

4. Analogia Fidei. A different meaning for the word analogy, at least when compared with its counterpart in Aristotelian-Thomistic philosophy, is that present in the expression "analogy of faith" (*analogia fidei*). It is first found in the letter of St. Paul to the Romans ("Let he who has the gift of prophecy make use of it according to the measure of faith," *Rom* 12:6), where the Greek term *analoghía* is used in the sense of "measure" or "proportion." In the Catholic tradition, this expression has taken on a technical character and signifies the inner coherence and harmony between the truths of faith that cannot contradict each other. The *Catechism of the Catholic Church* defines it today in the following way: "By 'analogy of faith' we mean the coherence of the truths of faith among themselves and within the whole plan of Revelation" (CCC 114). The analogy of faith guides us in our interpretation of the Old Testament in light of the New Testament. It is essential, indeed for a correct understanding of what the "development of dogma" means. Under the guidance of analogy, such development must not be viewed as a change in the content of truth but as the consistent deepening of understanding of the same revealed truth. Classic sources for this understanding can be found in St. Vincent of Lerins (cf. *Commonitorium*, 53: PL 50, 668) and in John Henry Newman (cf. *An Essay on the Development of Christian Doctrine*, 1845).

Reformed theologians, especially Karl Barth (1886-1968), made use of the expression *analogia fidei* to indicate the one and only source of knowledge about God, that of Divine Revelation, as opposed to *analogia entis* understood as the foundation of the path that allows natural reason to reach a non-revealed knowledge of God, a path that the Lutheran view rejects. Refusing the possibility that there could be an analogy-based knowledge of God arising from the experience of creatures, such theologians attempt to base the possibility and intelligibility of Revelation solely on the gift of grace. According to Karl Barth, "our human concepts and our human terms, in so far as they are ours and human, are totally incapable of expressing God and His mysteries; their aptitude for adequate and correct expression comes only from

revelation." One may say of God only what God says of Himself, that is, his Word, Christ. It should be observed, however, that such a perspective does not seem to solve in a convincing way the problem of how to ground the intelligibility and understanding of the revealed word, in that, even though we are helped by grace, our understanding of God is always expressed through our own words, which are the only words we have at our disposal. "It remains true that the notions chosen by Christ to introduce us to the divine mystery are still human notions. Christ borrowed them from human language, from the whole range of created realities. And it is on the basis of these realities, objects of human experience, that is effected a purification and development of meaning which are dictated by the necessities of revelation [...]. If Christ can utilize all the resources of the created universe to make us know God and the ways to God, it is because the word of creation has preceded and left a foundation for the word of revelation; it is because both one and the other have their principle in the same interior Word of God. The revelation of Christ presupposes the truth of analogy" (R. Latourelle, *Theology of Revelation* [New York: Alba House, 1966], pp. 366-367).

IV. Analogy and Science

Up until now, the concept of analogy has never been a part of any scientific theory, even though it has always in fact accompanied the progress of science from the outside, suggesting new avenues of research and new interpretations of results. This can be understood by considering the fact that modern science, which employs the Galilean method, is as mathematical as possible. In mathematics, as it has been developed up to now, every symbol used in the same proof must unambiguously correspond to a single definition. In the second place, even when direct use is not made of mathematics, univocity is systematically adopted so as to avoid the possibility of ambiguity or of error. It is, however, interesting to observe that in the last decades, research concerning the science of complexity and self-reference in different fields seems to demonstrate the theoretical limits of univocity and to suggest an analogy-based approach.

1. Analogy and Scientific Theory: The Experimental Sciences. The word "analogy" is often used by scientists in their qualitative descriptions of their results, even though it has never been a part of any scientific theory. In particular, analogies have proven to be useful throughout the history of science and have been used for a two-fold purpose: (a) to suggest a way to build a theory (a heuristic purpose), and; (b) to aid in interpreting an already developed theory which is similar to another theory because it has a similar mathematical structure (a hermeneutic or interpretative purpose). In both cases, analogy, however, does not play a direct part in the mathematical formulation of the theory, in that the symbols used continue to have an unambiguous definition. And, it must be emphasized that from the Aristotelian-Thomistic point of view, we are dealing with "analogies of proper proportionality," that is, with similarities between relations. These similarities lie at the root of any possible model describing certain facts of experience [2]. In particular, analogies, thus understood, can be said to be "material," i.e., concerned with the "physical structure" of the systems to be described, or "formal," i.e., concerned with the "mathematical laws" that describe and explain the determined behavior of physical systems.

"Material analogies" are useful in describing the properties of a system of which the internal structure is still unknown: One assumes that the unknown structure of the system might be similar to that of another well-known system and governed by a known law. In such cases, a "model" is proposed for the system to be to described. A familiar example, in physics, is provided by the model of "elastic rigid balls," which is adopted as an approximate description of the behavior of gas molecules. In instances such as these, the similarity between the model and the physical phenomenon is supposed on the level of the structure of the material components; consequently, it is also expected that there will be a behavior that is similar to both

systems, and similar laws supposed to govern both. This involves analogy of proper proportionality, which can be expressed by the following statement: “The rigid balls are to their dynamics as the molecules are to their own dynamics.” A similarity between the relationships (balls-dynamics and molecules-dynamics) is supposed, which is so tight as to legitimate the use of the same law to describe both systems within an acceptable margin of error.

On the other hand, “formal analogies” are not based on a model of the physical constituents of a certain system but on mathematical equations capable of describing its behavior without any hypothesis regarding the material structure governed by such laws (cf. Nagel, 1961). This way of proceeding is less natural to those who are not used to representing things in mathematical terms, whereas it is completely obvious to the mathematical physicist, accustomed to substituting the physical object in his or her mind with the mathematical equations that govern its behavior. In such cases, the similarity lies at the level of the “physical laws” governing the systems, which are supposed to be represented by the same equations within an acceptable range of error. In some cases, the formal equivalence of certain equations (which, however, have different physical interpretations of the same mathematical symbols) lead to new theories that are difficult to formulate without the aid of such a formal analogy. The most significant example of this is found in wave mechanics: The Schrödinger equation, which is the fundamental equation of Quantum mechanics [7], is obtained through an analogy between geometrical optics and classical analytical mechanics [8].

Aside from the heuristic aspect of analogy in the sciences, there is also a hermeneutic aspect. Analogy, in fact, can aid in the interpretation or explanation of the behavior of a system for which a certain model is adopted because it serves the purpose of reducing a lesser known phenomenon to a better known one. Suffice it to think of all of the microscopic models developed to explain the behavior of a macroscopic system: Kinetic theory, for example, gives, as a mechanical-statistical model of a thermodynamic macroscopic system, a detailed understanding of the macroscopic processes involving the state variables that characterize the system. In this case, the analogy which one forms is the following: “The kinetic model is to the laws of statistical mechanics as the thermodynamic system is to the laws of thermodynamics.” If we accept this analogy and assume that it is possible to identify the laws of kinetic theory with those of thermodynamics within an acceptable margin of error, we can obtain a relationship between the kinetic theory quantities and those of thermodynamics and thereby obtain a kinetic interpretation of the latter. One might think, for example, of the conceptual identification of the absolute thermodynamic temperature with the average translational kinetic energy of the molecules in a gas. In this case, analogy proves to be advantageous since it leads to a new understanding.

2. Analogy and Scientific Theory: The Mathematical Sciences. If in physics analogy does not play a direct role, except as a methodology that suggests from the outside how to build and interpret theories, formal analogy has a similar role in the development of new mathematical structures. The latter are intended to be based on simpler models for which one looks for a generalization that keeps some of their formal properties. It is important to keep in mind that in both physics and mathematics, analogy does not directly come into play as an “internal” element of the theoretical system but rather plays a role in the building and interpretation of science. It is true that in the internal structure of mathematics there are biunivocal relations between elements of distinct sets (isomorphisms, homeomorphisms, diffeomorphisms, etc.), but we are not dealing, in this case, with real analogies of proper proportionality in the sense above, but instead with structural identities. In these cases, there is a complete identification, and not only a similarity, between the relations. For this reason, such sets are indistinguishable as far as the properties of the structure are concerned, and it can be said that each of these sets is a “model” for the structure under consideration. In Aristotelian-Thomistic language, one could say that these models are like the “species” of the same “genus.” A well-known example is found in the so-called “Euclidean models” of

non-Euclidean geometries and, more generally, in any mathematical model with an abstract structure. A non-Euclidean geometry, for example, can be thought of as abstractly defined by its axioms, regardless of the fact that there are different realizations of any one of its models. Nevertheless, as soon as we realize these models, they are not simply analogous but completely isomorphic to each other. This is because every relation between the elements of the model corresponds to an identical, not just a similar, relation between the elements of the other model. In the example of non-Euclidean geometries, we might think of the hyperbolic geometry of Bolyai that can have as a Euclidean model the Klein model in the plane (cf. Courant and Robbins, 1996).

Another well-known example of two mathematical models with the same structure is found in quantum mechanics, which admits a two-fold representation in two isomorphic Hilbert spaces; that is, the Schrödinger picture, formulated in terms of wave-functions in an L2 Hilbert space (square integrable functions), and that of Heisenberg, expressed in terms of l2 vectors expanded on an orthonormal basis of eigenfunctions (cf. Fano, 1971).

3. Analogy within Scientific Theory. Interest in analogy and research devoted to the development of a "scientific theory of analogy" and a "method of demonstration" based on the latter, seem to emerge inevitably from the study of systems (whether they are biological, chemical, physical, mathematical, logical, etc.) that are organized according to "hierarchical levels." Some of these levels cannot be reduced to more elementary ones (cf. Cini, 1994) because they differ not only "quantitatively" but "qualitatively." They have different natures but, at the same time, something real in common. In this case, it seems possible and useful to invoke the analogy of simple proportion or that of proper proportionality.

Up until now, the sciences have involved the search for components that act as fundamental "parts" or "building blocks" to explain the structure of the universe as a "whole," assuming that the parts have the same nature as the whole (matter-radiation). In this scheme, the "building blocks" of the whole, according to the Standard Model, are "quarks" and the "gluons" that bind them, which form particles once believed to be elementary and which in turn form nuclei and atoms, which then form molecules, and finally, living cells and more complex living organisms. Every level of this scale is considered perfectly homogeneous with the other levels, made of the same matter, and considered of the same nature. In a sense that seems to contradict this way of framing the problem, qualitatively diversified (and, hence, irreducible to each other) levels have a tendency of emerging in the same system. If in fact one of these levels of organization (the "higher level") were in some way deconstructable to other, more elementary ones (the "lower levels"), and if it could be reconstructed through an appropriate reconstruction of the latter, the higher level would not be "qualitatively" different but a simple "superimposition" of the lower levels. These different levels do not represent absolutely disparate properties that cannot be compared to each other, but constitute, instead, different ways of manifesting and realizing the same property, which can therefore be actuated in varying ways (that is, not univocally), but according to differentiated ways which are really related to each other (that is, analogically). In particular, we are faced with a two-fold modality in the relationship between the whole and its parts. On the one hand, we have a whole that is not reducible to the sum of its parts but possesses a new informative and unifying element that characterizes it as a whole. On the other hand, we have parts in which there exists something similar to the whole. Scientists commonly describe such a structure as "complex" (cf. Nicolis and Prigogine, 1989).

This situation is encountered today in every scientific discipline: The irreducibility of the levels is none other than a sign of the insufficiency of reductionism [9] in formulating scientific theories that deal with complex systems (cf. Dalla Porta Xydias, 1997). The biological sciences, for example, have always dealt with properties of living beings that are not shared by non-living beings, even from the chemical and physical point of view. The behavior of a living being, even the simplest, cannot be described entirely by

its constituent parts. On this level, the analysis of the constituent parts is no longer enough, and a study of the new level of the whole is necessary. A thorough study of a somewhat complex molecule, such as those found in a crystal lattice, or a study of the impurities in a crystal that determine the electrical properties of an entire semiconductor, to cite a few examples, have shown that even in the chemistry of non-living objects, the properties of the whole of a complex, composite structure cannot be deduced from the properties of the atoms that comprise it. The existence of molecular orbitals of fully shared electrons no longer allows us to think of those electrons as belonging to a single atom. In an electric conductor, the conduction electrons are in fact shared among all the atoms of the lattice. In the fields of physics and mathematics, the problem of the whole and of the parts is clearly of relevance in the two senses alluded to above: In particular, the "non-reducibility of the whole to the sum of the parts" is a consequence of the "non-linearity" of the differential equations that govern complex physical systems, whereas the self-replication of the whole in each of its parts is none other than a sign of "self-reference," which is of great relevance to the logician and to the computer scientist. In fact, it seems that computer scientists were the ones to revive the by now classical problems of mathematical logic. Take, for instance, the problems related to Godel's theorem concerning the consistency and completeness of axiomatic systems, or the problem of displaying fractal sets, in all their beauty [10], on the computer screen, which up to then had seemed to be "mathematical monsters" due to their infinitely winding boundary (as the Julia sets). Benôit Mandelbrot's work served to rekindle interest in these problems. The field of fractal geometry began to develop when computers were utilized as laboratories in which mathematical experiments could be performed, in a way similar to the manner in which Archimedes, more than two thousand years ago, performed mechanical experiments so as to catch a glimpse of geometrical properties; only later would he seek a logical demonstration of such properties beginning with a set of axioms. Research in the field of artificial intelligence, in addition, has afforded an understanding of the fact that information can be found on various levels and that there can be different hierarchies of information [11]. The lower level lies in the hardware of the machine, and the higher levels in the software. The programming language, in turn, contains the higher-level information that is meaningful for the programmer, which implies, in turn, lower level instructions mechanically executable by the machine, which cannot perceive their higher-level significance.

The program itself, as a whole, involves higher-level information related to the goal for which it was written (which lies in the mind of the programmer and in that of the user, and so on and so forth). In every scientific discipline, there seems to be a hierarchical structure of information related to the degree of complexity, and therefore of the unity of the structure studied. It therefore seems necessary to widen the scope of current scientific methodology and rationality so that the sciences can overcome the barriers erected by impossibility theorems such as that of Gödel (cf. De Giorgi et al., 1995).

The need for such a widening of scope is felt, first of all, in the study of "non-linearity." From the mathematical point of view, and therefore from the point of view of all mathematical sciences, the impossibility of conceiving the whole as the sum of parts that are homogeneous with the whole (reductionism) is encountered in the field of non-linear differential equations for which, as it is well known, the sum of two or more solutions is not a solution, and conversely, for which every solution cannot be written as a linear combination of simpler solutions (which is the case with linear differential equations). Therefore, it is not possible, in general, to reduce the study of any given solution to simpler and already determined solutions in a non-linear system. Moreover, nature herself is described in great part by systems of non-linear equations, and linear solutions are only a first approximation. Non-linearity, therefore, introduces the concept of the "irreducibility" of certain solutions to simpler ones. The different solutions, however, have something in common: They are all solutions of the same equation.

In the second place, the problem of self-reference must be considered. By "self-referring," a term

originating in the field of logic but which is now universally used, one means an operation or system whose "whole" is completely replicated, i.e., is completely identical to itself, in its parts. Self-reference was discovered by the logicians of Ancient Greece who viewed it as a possible source of contradictions: One thinks of the famous "liar's paradox" in its varying versions. For the same reason, modern logicians and mathematicians have carefully kept self-reference out of their axiomatic systems. Betrand Russell (1903) excluded it from his set theory, where it had emerged, for example, in the idea of the "self-inclusion" of certain sets of elements, which contain themselves. Kurt Gödel (1931) had succeeded, on the contrary, in exploiting precisely the possibility of creating paradoxes through self-reference for the purpose of proving the non-decidability of certain propositions of formal systems, such as the *Principia Mathematica*. He deduced the incompleteness of such a system and the impossibility of demonstrating its consistency from within the system. The use of the computer, which makes wide use of recursive algorithms, once again brought up the problem of self-reference in the fields of logic and mathematics. If it is clear that self-reference can lead to contradictions, it is likewise just as clear that this does not always, and does not necessarily, happen. We have a contradictory self-referring proposition when the predicate negates the truth of the proposition itself. For example: "This proposition is not true." In like manner, we have a contradiction in set theory when we restrict the set of all sets not to include itself: "The set of all sets that do not contain themselves" is contradictory because the definition implies the set contains itself and does not contain itself at the same time. Nevertheless, certain contradictions can be avoided if one has a clear idea as to how self-reference can be applied to "differentiated levels" of the same object, and if one understands that it must be interpreted in an analogous, and not a univocal, sense. In this case, the "whole" cannot replicate into copies that are "identical to itself" but only "similar to itself."

4. The First Steps towards a Theory of Analogy. In this subsection, I will set forth a few examples. The first example involves acknowledging a hierarchy of levels. Where does the contradiction lie in the self-referential proposition, "This proposition is not true," or in the definition of the "set of all sets which do not contain themselves"? The contradiction arises because the "proposition" ("this proposition is not true") and the subject "this proposition" are identified with one another, whereas, in reality, they are not the same proposition. They share the fact of being propositions in common, but they differ in the "manner" in which they are propositions. Likewise, the "set of all sets which do not contain themselves" is not a set in the same manner as the "sets which do not contain themselves." The fact of identifying them (univocity) does not take into account the difference in the mode of being of the sets and therefore gives rise to the contradiction. In order to eliminate this contradiction at its root, Russell proposed clasifying the sets into "sets of differentiated types." Sets of simple elements (that is, elements which cannot themselves be sets) belong to the first level (or type). Sets whose elements can only be sets of the first type belong to the second level (or type). Sets of the third type are those whose elements are sets of the second type, and so on and so forth. In this manner, one obtains a hierarchy of sets belonging to different well-defined levels. Thus, the term "set" can be said in different senses depending on whether or not one is speaking of sets of the first, of the second, or of another level. In a similar manner Gödel proposed a solution to the paradox of the universal class (the class of all sets) by distinguishing two types of classes: the "proper" ones that, by definition, are not allowed to be contained in wider classes, and the " improper "classes (or sets) that may belong to a wider class. According to this, two different ways of being a class, both the universal class and the Russell class, result in being a proper class and they are no longer paradoxical (Gödel [1938], 1990, p. 38).

A similar classification is made for propositions. To summarize, we can say that one has made the first small step towards the concept of analogy due to a need arising from within the system. And, this first step consists of introducing levels, or differentiated ways, according to which the same term can be predicated, and the same object can exist, as, in our case, a set or a proposition. It must be observed in this

kind of analogy that it is possible to establish similarities between relations of different types of sets, in a way similar to what happens in the analogy of proper proportionality.

Connected to the topic of self-reference, another important direction can be found in the field of fractal geometry. Fractals are geometrical structures that often have the noteworthy property of being "self-similar," that is, they replicate themselves infinitely in each of their parts. In certain cases, as the curve of von Koch, such self-similarity is so perfect that it is impossible to determine the scale of magnification of a given level, since the replicated form is always the same in every part (cf. Peitgen and Richter, 1986). In other cases, such as the Mandelbrot set, there is not a complete self-similarity, but an infinite replication of itself into "similar" copies that are not exactly the same as the whole. Unlike what happens with sets or propositions, each of the parts of a fractal that replicate the whole are not, however, identical to the whole. But, though being distinct from the whole, it is nevertheless similar in form to it. In this case, it is preferable to speak of "self-reference" instead of "self-referentiality." The latter geometrical example, even if it only gives a geometrical representation and is only an informal model, allows us to make a few considerations: (a) The geometrical structure is "similar" in its whole and in its parts, even if such a structure is actualized in slightly different ways in each part. Therefore, one cannot speak of complete identity, but only of similarity, as it so happens in the analogy of terms; (b) Every replicate is not properly speaking separable from the whole, but always subsists as a part of the primary whole. For this reason, the whole can be compared to a sort of "*analogatum primum*" (as in the "analogy of proportion") on which every part physically depends, and; (c) One can establish relational correspondences between the parts and the whole, and among the parts with each other, as in the "analogy of proportionality."

A further step can be made if we acknowledge the difference between "essence" and "existence." The decisive leap, which is needed for analogy in the strict sense, is to begin thinking of "objects" (as the scientist would say) or of "entities" (as the philosopher would say) that are "similar" but irreducible to the same "mode of existence." In order to characterize different "modes of existence," one needs to avoid reducing existence to a simple logical "non-contradiction," as is the tendency in formal logic. This kind of reduction makes the very notion of existence univocal, as it postulates that which is not contradictory, that is, that which is thinkable, exists, and exists only because it is not contradictory and only according to a single mode determined by its non-contradictory nature. In philosophical language, this position is equivalent to that of "the identity of essence and existence." Gödel's theorem has shown this kind of mathematical approach to be insufficient. The first attempt to refute mathematical formalism through the distinction between existence and essence can be found in the intuitionistic approach (cf. Basti and Perrone, 1996). The intuitionistic approach goes to an extreme position that denies the universal role of essence and overemphasizes that of existence. In fact, intuitionism posits the distinction between essence and existence by denying the "principle of the excluded middle": In this way, proofs by contradiction are insufficient to prove the existence of a mathematical entity and are only capable of showing its logical impossibility. Existence must be proved with a constructive, finite method. Only what can be constructed with a finite number of operations exists. In other words, only this or that particular model can be constructed, and therefore the universal cannot be reached and remains a pure name (Nominalism). It is interesting to observe how both formalism and intuitionism assume a univocist mind-set, whereas the analogy-based solution, which acknowledges differentiated modes of existence of the universal and the particular, seems to be more appropriate (cf. Ibid., pp. 220-223). Research in this direction is still in the developmental phase.

Another scientific field in which the concept of analogy is being used is that of artificial intelligence [12], or better yet (and more generally), that of cognitive science, a wider field of study which involves not only problems dealing with machine learning but more generally problems in psychology, such as the

mind-body relationship and the relationship between the mind and the brain (see Mind-body relationship) [13]. It is important to stress the effort made to overcome Cartesian dualism, a philosophical position according to which the mind and body are two separate "objects" joined together in a completely extrinsic way (cf. Basti, 1991, p. 105). On the one hand, computer science has in practice forced the revision of such a dualistic-mechanistic view. In fact, information inserted in a machine by means of software and input peripherals, which allows the machine to interact with the external world, is not a "thing" to be placed on the same footing as the hardware, but lies on a higher plane. The stratification of different levels of information allows one to establish relationships between entities of different levels (which recalls the analogy of proportion) and relations between these relationships (which recalls the analogy of proportionality). In this way, a structure of information emerges which is in a certain sense analogous. On the other hand, the experimental study of the mind-body relationship of human cognitive processes has convinced several scientists that the human mind works by analogy and not simply through an accumulation or extraction of information from a kind of data base (cf. Hofstadter et al., 1998). Consequently, with the aim of imitating human intelligence with a computer, a way is sought to reproduce this kind of analogy-based operation rather than simply a way to store a lot of specific information concerning the problem that the machine is to solve according to a reductionist mind-set that isolates single parts of an object from all the rest. Certainly, it is not enough to found a theory on a merely intuitive notion of analogy taken from its everyday meaning in common language. A rigorous theory of analogy is therefore needed.

V. The "Profundity" of Analogy

In conclusion, the genius of analogy, about which scientific interest is gradually increasing, lies in two fundamental aspects: (a) the fact that it distinguishes between qualitatively different, but really related, levels of the same entity; (b) the fact that it is inseparable from a true extra-mental reality that participates in the being. The Aristotelian-Thomistic concept of analogy, as we have striven to point out, acknowledges different hierarchical levels of being that differ by their very nature. For this reason, there are "things" and "principles" that allow these things "to be" and "to be what they are." The "principles" and "things" are irreducible to each other for the very reason that they have different natures. At the same time, they are not completely heterogeneous with one another since they constitute different modes of the same being they possess in a differentiated way. In Latin terminology, *quod* indicates the "thing" and *quo* the principles by which the thing "is" and "is what it is," that is, they possess their own characterizing properties. In the language of modern physics, we would say that that which is "observable" is a *quod*, whereas the *quo* is not only unobservable in practice, since it is in a certain sense confined in virtue of a certain infinite potential barrier (as a quark in an infinitely deep potential well), but it is also not observable due to a theoretical reason, since it is of a completely different nature from the observable. For example, if the "thing" is a particle, its constitutive "principle" is not a particle, or at least not in the same way, but in an analogous way. For this reason, the "principle" is not observable. The unobservable *quo* is introduced, not as a superfluous element of the theory (as if it were a hidden variable that could be eliminated), but as a simple principle which is in a certain sense necessary and inevitable in order to account for the observable phenomenon. It is clear that the mathematical sciences, in their current version, are not yet in a position to introduce into their language a *quo* that is irreducible by nature to a quantitative and relational *quod*. Nevertheless, in a "broad enough theory," such an introduction seems possible and plausible. In this way, one can broaden a reductionist theory to a non-reductionist one that is able to accommodate principles that are irreducible and analogous to each other, without falling short of the demands for the rigor of a formal theory.

The second characteristic that we cannot afford to ignore in the theory of analogy is the close tie between

logic and truth [14], or in other words, the relationship between, on the one side what is thought and, on the other side, what is extra-mental reality. Analogy can be fully understood only if it is considered a logical description of what does happen in the extra-mental reality of things, since it is capable of describing on the logical level what is a reality on the ontological level. Consequently, a broad theory with which one can formalize analogy in the sense understood here must be able to accommodate the distinction between both a purely logical-formal mode of existence (non-contradiction) and different real modes of existence (extra-mental) through the distinction between essence and existence.

Analogy is one of the tools that allow us to understand why essence and existence are not reducible to each other. In a certain way, it constitutes a response to the incompleteness of existential philosophy (the truth of the thing leads only to its emergence in the stream of existence and not to other questions) and of the essentialist philosophy (the truth of the thing consists only of the explanation of what it is, that is, its essence). Analogy also serves as a guide aiding us in the correct use of language and symbols as it prevents language from ending up in a continuous regress with no epistemological basis.

Read also: Experience [2]
Laws of Nature [15]
Mechanics [8]
Truth [14]
Documents of the Catholic Church related to the subject:
Abbreviations and complete titles of the documents [16]

Divino Afflante Spiritu, DH 3826 [17]; *Fides et Ratio*, 19 [18]; *Humani Generis*, DH 3887; *Dei Verbum*, 12 [19]; *Lateran Council IV*, DH 806; *Providentissimus Deus*, DH 3283 [20]; *Vatican Council I*, DH 3016 [21].

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Translator: Chang

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